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A theorem of I. D. Ado asserts that a finite-dimensional Lie algebra over a field of characteristic 0 is isomorphic to a linear algebra over $\mathbb{K}$. This theorem, first given by Ado [Bull. Soc. Phys.-Math. Kazan (3) **7** (1934/35), 3–43; JFM 61.1217.02], and proved rigorously by E. Cartan [J. Math. Pures Appl. (9) **17** (1938), 1–12; Zbl 0018.14701], was actually introduced in a number of books on Lie theory with proofs attributed to K. Iwasawa (whose proof is still valid for the case of positive characteristic) [Jap. J. Math. **19** (1948), 405–426; MR0032613].

Let L be a finite-dimensional Lie algebra over the field $\mathbb{K}$, which may be of any characteristic. A subalgebra S of L is called subnormal if there exists a chain of subalgebras $S \subset S_1 \subset S_2 \subset S_r = L$, each an ideal in the next member. In this paper the author proves the existence of a finite-dimensional L-module V which gives a faithful representation ρ of L which has the extra properties:

1. V is $S\mathfrak{F}$-hypercentral for every saturated formation $\mathfrak{F}$ and every subalgebra subnormal S of L with $S \in \mathfrak{F}$.
2. The action $\rho(x)$ of x on V is nilpotent for every $x \in L$ with $\mathrm{ad}(x)$ nilpotent.

In particular, if f is supersoluble (i.e. if it has a sequence $S \subset S_1 \subset S_2 \subset S_n = L$ of ideals in L with $\dim(A_i/A_{i-1}) = 1$ for all i) and S is a subnormal subalgebra of L, then L has a faithful finite-dimensional representation in which S is represented by upper triangular matrices. *José María Ancochea Bermúdez*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.