A theorem of I. D. Ado asserts that a finite-dimensional Lie algebra over a field of characteristic 0 is isomorphic to a linear algebra over $K$. This theorem, first given by Ado [Bull. Soc. Phys.-Math. Kazan (3) 7 (1934/35), 3–43; JFM 61.1217.02], and proved rigorously by E. Cartan [J. Math. Pures Appl. (9) 17 (1938), 1–12; Zbl 0018.14701], was actually introduced in a number of books on Lie theory with proofs attributed to K. Iwasawa (whose proof is still valid for the case of positive characteristic) [Jap. J. Math. 19 (1948), 405–426; MR0032613].

Let $L$ be a finite-dimensional Lie algebra over the field $K$, which may be of any characteristic. A subalgebra $S$ of $L$ is called subnormal if there exists a chain of subalgebras $S \subset S_1 \subset S_2 \subset \cdots \subset S_r = L$, each an ideal in the next member. In this paper the author proves the existence of a finite-dimensional $L$-module $V$ which gives a faithful representation $\rho$ of $L$ which has the extra properties:

1. $V$ is $S\mathcal{F}$-hypercentral for every saturated formation $\mathcal{F}$ and every subalgebra subnormal $S$ of $L$ with $S \in \mathcal{F}$.

2. The action $\rho(x)$ of $x$ on $V$ is nilpotent for every $x \in L$ with $\text{ad}(x)$ nilpotent.

In particular, if $f$ is supersoluble (i.e. if it has a sequence $S \subset S_1 \subset S_2 \subset S_n = L$ of ideals in $L$ with $\dim(A_i/A_{i-1}) = 1$ for all $i$) and $S$ is a subnormal subalgebra of $L$, then $L$ has a faithful finite-dimensional representation in which $S$ is represented by upper triangular matrices.

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References

11. H. Strade and R. Farnsteiner, Modular Lie algebras and their representations (Marcel
Dekker, New York, 1988). MR0929682

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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